Fen Rivers Academy

Misconceptions in Mathematics

Preface

What are misconceptions in mathematics?

Misconceptions in mathematics are a natural consequence of a child's mathematical development. It is important to emphasise right from the start that misconceptions are not a negative aspect to teaching or learning. On the contrary, they often reveal much about children's thinking and how they acquire, or not, an understanding of complex mathematical concepts.

Misconceptions can often be used as useful discussion points in lessons as they can highlight how pupils are thinking about specific areas of mathematics.

How might they arise?

Misconceptions can arise because children learn mathematics through constructing their own knowledge and understanding. As children construct their own meaning it is inevitable that they will make mistakes. It is part of the learning process.

Mathematical understanding centres on children first having concrete, practical experiences before it becomes an abstract subject in which they need an understanding of symbols and how they are connected. If children miss out some of these stages, or are not secure enough in one stage before they move to another, they may transfer their knowledge inaccurately, thus developing a misconception. For further guidance on the importance of using manipulatives and resources please refer to the Representations in Mathematics guidance document.

Children also learn through interactions with their peers. Sometimes these interactions can cause mathematical misconceptions. As the children communicate their thoughts and ideas to each other, misconceptions may arise that are then transferred from one child to the next. In this situation, children can be exposed to inaccurate mathematical ideas that then form their own beliefs about mathematics. In particular, children often use incorrect language, which can subsequently lead to misconceptions developing.

Finally misconceptions may arise because children are taught the mathematics incorrectly. It is important that teachers are aware of the potential misconceptions, they are skilful in selecting the activities to match the learning objectives and that they provide clear explanations when teaching and modelling mathematics to the children. However, even if teachers are highly competent, children will still form mathematical misconceptions.

How can we tackle them?

The most successful way in tackling misconceptions in mathematics is to be aware of them, and to help children to overcome them when they naturally arise. In addition, when we are aware of the potential errors that children may make, we can plan appropriately to avoid explicitly teaching a misconception (for example, not telling children that when we multiply by ten, we add a zero). Understanding how children construct their mathematical knowledge allows us to plan appropriately for the errors that they make and use these to enhance children's learning.

We need to ensure that correct mathematical language is used. Simple misconceptions can be avoided if both teachers and children use and understand mathematical vocabulary. Many of the words we use in mathematics also have everyday meanings too. For example, the word face can refer to faces on people but in mathematics refers to the properties of shapes and how we describe them. It is important to discuss both types of meanings with the children so that they become aware of both interpretation of words and can recognise their mathematical meaning in mathematics lessons. For further guidance on using correct mathematical vocabulary, please refer to the mathematical vocabulary and glossary guidance document.

It is also important to tackle misconceptions by explicitly teaching them too. We know that even with the best teaching, children will still make misconceptions in mathematics. Therefore, although we can minimise the amount children make, it is still valuable to view the teaching of misconceptions as an opportunity to assist further learning. Using misconceptions in this way is to teach children the importance of learning from mistakes. However, it is not enough for teachers to simply explain the misconception. Children will also need help in understanding the correct mathematical concepts too. Teaching approaches which encourage the exploration of misconceptions through discussions result in deeper, longer term understanding than approaches which try to avoid them by explaining the 'right way' from the start.

In addition to this, if children become more familiar in learning from and understanding the value of misconceptions, it is hoped that they may feel more open about sharing some of their own. This approach is based on the belief that children's mathematical understanding is developed through children explaining their thinking to the teacher and also to other children. In this way, mathematical misconceptions are necessary steps in children's learning.

In summary, teachers need to be aware of mathematical misconceptions, understand how they arise, understand how they can be prevented, and be able to solve them when they appear.

Finally, it is important to distinguish between mistakes and misconceptions in mathematics. Your response to dealing with mistakes compared to misconceptions will be different. A mistake could be made for several reasons. For example, it may be a misinterpretation of symbols or numbers or a simple calculation error. However, a misconception could occur because children do not understand that when we multiply by ten we do not just add a zero, for example. Both mistakes and misconceptions can be made by all children – not just those children who appear to need more help in mathematics. This guidance document only highlights mathematical misconceptions, and not the everyday mistakes that children may make.

How to use this booklet

This booklet contains many of the common mathematical misconceptions that children will come across, but there are more! It is hoped that through becoming familiar with the ones highlighted in this booklet, you will be able to plan for and tackle any other misconceptions that may arise in your teaching.

You should not assume that just because a misconception is not included in this booklet, this means that children do not make errors related to it. It is hoped that by using this booklet, and becoming much more aware of the mistakes and misconceptions children make, you will be able to identify and tackle many more.

The misconceptions included in this booklet are known difficulties for children. They are organised under five mathematical areas:

- numbers and operations;
- algebra;
- measurement;
- geometry;
- statistics.

Each misconception is explained, an example given and then a possible solution offered. Where possible, solutions focus on using concrete, physical models for children to experience and understand the mathematical concepts. Other solutions encourage discussion and explanations between both children and teachers.

Contents

Numbers and operations

This section has two main parts to it: numbers and operations. To make it easier to follow, the first section on numbers is split into whole numbers and fractions, decimals and percentages.

Whole Numbers

Misconception: inaccurate counting.
Example: A child counts five objects as ‘One, two, three, four, six, seven’.
Solution: There are two misconceptions here. First the child has counted an object more than once, and second, they have missed out five in their counting sequence. Model how to count using one to one correspondence; for example, touch an object when counting it and move it away from the group. Also, give lots of experience in counting aloud in number sequences, so that children hear the number names in order.

Misconception: including the starting number when counting.
Example: there are three objects to count, but children start at zero and count ‘zero, one two’ so the cardinal value is wrong.
Solution: use a physical model such as a number line. Place a counter on zero. Now ask children to count on three. Show the children that to count on one, we move the counter to the number one, then two and then three.

Misconception: the position of a digit relates to its value.
Example: in the number 505, children are confused about the value of the two fives. Children can also mis-read the number 208 as twenty eight, not recognising that the two is worth two hundreds because it is in the hundreds column. Conversely, when asked to write six hundred and nine, a child may write 6009 (i.e. 600 + 9). Again this error occurs because the child does not understand that the position of a digit determines its value.
Solution: use place value charts and cards to show children that the value of the digit is determined by which place it is in.

Misconception: written numbers do not always appear as they are spoken.
Example: the number 14 is written as the number 41
Solution: this is a common misconception among young children who are trying to make connections between spoken and written numbers. Use place value charts and cards so they can see the difference between 14 and 41. Talk about how the four is always said first even though this it is not written first in one of the numbers. It is important that children understand the structure of how numbers are made up so that they realise there are potential misconceptions out there.

Misconception: rounding numbers inaccurately.
Example: 14 493 rounded to the nearest 1000 is 15 000.
Solution: If children are used to rounding numbers they could have rounded 14 493 to 14 490 for the nearest 10, then 14 490 to 14 500 for the nearest 100, and 14 500 to 15 000 for the nearest 1000. However, the correct answer for 14 493 to the nearest 1000 is 14 000. Show children the difference between 14 493 and 14 000 is 493 and between 14 493 and 15 000 is 511. Alternatively, use a number line and show the difference between the two numbers visually.

Misconception: zero as a quantity.
Example: when asked how old her sister was, Jennifer replied ‘zero’.
Solution: talk to children about the different representations zero can have. We often use it to represent nothing, but it is also a number. In the example above, Jennifer’s sister is not zero but three months old. However, Jennifer could not translate her sister’s age into 0.3 years old and instead thought of the number that was one less than one.

Misconception: the concept of zero as a place holder.
Example: a child is working with base 10 equipment. They have 1000 and 34 cubes but the child writes 134. The pupil does not understand that if we have no hundreds, we must write zero in the hundreds column so that the recorded digits are given in the correct position; i.e. the one in the thousands position, the three in the tens position and the four in the units position.
Solution: using place value cards to show children that if zero did not occupy the place of the tens digit we may confuse the number 208 with the number 28.

Misconception: reading negative numbers
Example: children read -13 as 13.
Solution: This misconception could be due to either children not recognising the number as a negative number, or not understanding that negative numbers exist. Discuss with children that when we write or read negative numbers, the minus sign is usually placed in a raised position, to avoid confusion with subtraction. Use number lines where the zero is in the middle and negative and positive numbers are shown either side. Give children lots of examples where negative numbers are used. For example, temperatures that fall below zero, undersea divers or lifts that go below ground level.

Fractions, decimals and percentages

Misconception: you can’t have a fraction that is bigger than one.
Example: children do not recognise that 5/4 is a fraction and believe it is incorrect.
Solution: it is important to discuss with children that fractions can appear in many forms. To explain why you can have fractions bigger than one, use a physical example, e.g. two cakes. Cut both of the cakes into quarters. Give five of the pieces to one child. Ask how many quarters they have? Establish they have 5/4’s.

Misconception: when dividing fractions, parts are not always represented equally.
Example: a teacher asks the children to divide a semicircle into quarters. One pupil divides the shape with uneven quarters drawn on it.
Solution: while they have divided the shape into four, they have misunderstood that each part must be equal. Ensure that children realise that asking for ‘the biggest half’ of the chocolate cake is mathematically incorrect, although it is a common phrase used! Use lots of practical experiences when working with fractions so that children can physically compare each part and check they are equal. Also, make sure that different shapes are used, so that children do not become accustomed to regular or similar shapes.

Misconception: fractions as part of a set of objects are inaccurately interpreted.
Example: a group of objects are placed on a table (e.g. 3 blue cubes and one red cube). The teacher asks the pupils what fraction of the set is red. A pupil says ⅓. The pupil has not recognised the set of objects as a whole. They have compared the blue cubes to the red cube.

Solution: ask children how many objects there are on the table altogether (4). Explain this represents the total number of equal parts in the group and is the bottom number (the denominator) of the fraction. There is only one red cube so this represents the number of parts being considered (called the numerator in a fraction). The answer is ¼.

Misconception: fractions are only represented in one way.
Example: a fraction such as ¾ is always seen as 3 lots of 1/4 without recognition that it can also be a 1/4 of 3.
Solution: discuss with the children that there are many ways of representing fractions. Children are often presented with the first diagram to represent the fraction ¾. However, tell the children that the second diagram represents three pizzas that three friends have ordered. The first pizza is cheese and tomato, the second pizza pepperoni and the third pizza mushroom. Each friend would like a quarter of each pizza. They have eaten a total of ¾ of pizza but not from one pizza, but three, thus eating 3 lots of ¼.

Misconception: fractions are recognised as parts of objects but not numbers in their own right.
Example: the teacher is using the counting stick. One end of the stick represents zero, and the other end of the stick represents 10. She asks a pupil where the number ½ would go on the counting stick. The child counts to half way along the counting stick, pointing to five, instead of halfway between zero and one.
Solution: when children are introduced to fractions they begin with unit fractions such as one half or one quarter. They sometimes believe that a fraction is a number smaller than one (i.e. between zero and one). It is important to talk about fractions as numbers as well as parts of objects. In this example, the teacher is asking where the number half would be on the number line, not what half of the number line is.

Misconception: fractions with larger denominators are bigger than fractions with smaller denominators
Example: ⅓ is bigger than ½
Solution: use two piles of 12 counters. Give ½ of the counters from the first pile to one child (6), and ⅓ of the counters from the second pile to another child (4). Ask ‘Who has the most counters in their hand?’ Establish that the child who has ½ of the counters has more than the child who has ⅓ of the counters.

Misconception: to add two fractions together, you add across the top and the bottom
Example: 1/2 +1/4 =2/6.
Solution: Use a bar of chocolate where ½ is given to one child and ¼ is given to another. Ask the children how much has been given away (3/4). The difference between the two answers creates conflict and highlights the misconception.

Misconception: reading decimal numbers inaccurately.
Example: the decimal 8.76 is read as 876.
Solution: discuss the significance of the decimal point in the place value system. Look at place value charts and cards to understand what each digit represents in the number 8.76. Discuss with the children where 876 would go on a number line. Now look where 8.76 fits on the same number line. Show children that these are two different numbers, designated by the value of their digits.

Misconception: ordering decimal numbers.
Example: when given two decimal numbers, for example, 8.24 and 8.4, children believe that 8.24 is the largest number. When asked why, they say that 8.24 is larger

because 24 is larger than 4, so 8.24 must be larger than 8.4. The children believe that numbers are larger if there are more decimal numbers.
Solution: always ensure that children read numbers correctly. For example, 8.24 is read as eight point two four, and not eight point twenty four. Point out that for each place to the right of the decimal point, the numbers are successively smaller by powers of ten. Use decimal place value charts to show children the pattern of numbers as they decrease in size.

Misconception: doubling decimals.
Example: the children are asked to double decimal numbers (e.g. 0.52, 0.72). The children double all of the numbers but do not take into account that they are decimal numbers and give the answers 0.104 (instead of 1.04) and 0.144 (instead of 1.44).
Solution: the children have over-generalised the doubling rule which can be applied to whole numbers but they have not taken into account the decimal value of each number. For example, they have calculated that 72 add 72 equals 144, but not realised that 0.72 add 0.72 equals one point four four.

Misconception: multiplying decimals.
Example: 1 x 1 = 1 so 0.1 x 0.1 = 0.1.
Solution: Ask children to think of 0.1 as 1/10, so the calculation is 1/10 x 1/10. Remind children that the word 'times' is the same as 'of', so 1/10 times 1/10 can also be seen as 1/10 of 1/10, which is 1/100 or 0.01. 'One Squares' are a useful model to show children this visually too.

Misconception: the purpose of zero in decimal numbers.
Example: a child believes that 0.6 is smaller than 0.600.
Solution: in this example it could be that the child does not understand that the zeros in the hundredths and thousandths positions have no significance or that they have read the two numbers as 6 and 600. Use place value charts and cards to support the reading and understanding of decimals numbers. Discuss what the digits to the right of the decimal point are worth in value and explain that when zeros are used in the example of 0.600, the zeros have no value. It is also important to look at examples where the zero is holding the place, for example, in the number 0.04 or 0.308.

Misconception: percentages are always out of 100 objects.
Example: children are asked to colour in a 1-100 hundred square. They colour in 50 squares which is equivalent to 50%. The following day they are asked to colour in a number square that starts and one and ends at 80. They colour in 50 squares, believing they have coloured in 50%.
Solution: while percent means 'for each hundred' percentages are not always linked to 100 objects. We use the concept of per cent to describe a proportion of a quantity of a set. In the example above, there are 80 squares. It is useful to discuss other ways we can recognise 50%, such as ½ or 0.5. Recognising equivalent numbers will help the children to recognise 50% as ½ and not as 50 objects, thus helping them to recognise that 40 squares is 50% of the total set. In a different example, where there are 300 pupils in a school and 180 of them are girls, we can describe the proportion of girls as 60% of the school population. This means that there are 60 girls for each 100 pupils.

Misconception: percentages can never be bigger than 100%
Example: children hear their teachers saying they must give 110% today in their work. They are confused by this phrase.
Solution: since 100% represents the whole quantity being considered, it does seem odd to talk about percentages greater than 100. However, we can still correctly use percentages greater than 100 when we compare two quantities together. In these

examples, we are not expressing a proportion of a whole unit, but we are comparing two or more quantities. It is easier to give an example of a comparison. For example, if a person earns £50 in January and £60 in February, then we could record that the February earnings were 120% of January's. This is equivalent to saying one and a fifth of January's earnings. However, the phrase above is mathematically incorrect (and therefore a misconception) as it seems that we have earned more than we actually have.

Operations

Misconception: inappropriate use of the word 'makes'.
Example: two add three makes five (2 + 3 = 5).
Solution: children often interpret the equals sign as an instruction to do something with some numbers and to perform an operation. In the example above, children see the 2 + 3 and respond by doing something; i.e. adding the two and the three to get five. They interpret this as two and three make five. Talk to children about the equals sign as being an image for a pair of balance scales. Show the children that if we put five cubes in one side, the balance is not equal, unless we also put five cubes in the other side too.

Misconception: subtraction is only seen as taking away, and not finding the difference.
Example: in the calculation 18 - 4 = ?, children recognise this as 18 take away four, but do not also see it as what is the difference between 18 and four.
Solution: recognising subtraction as taking away is often the most frequently used structure for subtraction by both adults and children. It is strongly associated with the language of 'taking away' or 'how much is left?' However, finding the difference requires children to make a comparison between two quantities. They can do this by either taking the smaller number away from the larger number, or by counting up to the larger number from the smaller one. This can sometimes confuse children, because they associate counting up as an addition structure and not a subtraction one. It is important that children are taught to understand the operation of subtraction as both taking away and comparison. Give children subtraction questions and ask them to work out the answers first by using the 'taking away' structure and then by using the 'finding the difference' one. This will help children to understand that both structures result in the same answer. It is also useful to use concrete objects, such as cubes to demonstrate this. Build one tower of eight cubes and another of five cubes. Ask children to find the difference between the two towers. With a practical example, children naturally compare the two numbers, starting with the smaller one (five) and counting up to the bigger one (eight), resulting in the answer three. They have used the strategy of 'finding the difference' to work out the answer. Now ask children to record this calculation as a subtraction calculation (8 – 5 = 3). Use examples like this to show children that subtraction has different structures within it and how it is linked to addition.

Misconception: the commutative law applies to all of the operations.
Example: while addition and multiplication can be done in any order, subtraction and division cannot. 2 + 3 = 3 + 2 is correct; 3 - 2 = 2 - 3 is incorrect; 3 x 4 = 4 x 3 is correct; 12 ÷ 4 = 4 ÷ 12 is incorrect.
Solution: it is important for children to understand that the commutative law applies to addition and multiplication, but not subtraction and division. Use physical objects, such as beads, to show children that whichever we combine two or more groups for addition, the answer will always be the same. The commutative law is particularly helpful for children in understanding multiplication as it enables us to simplify some

calculations and also cuts down the number of multiplication tables we need to learn. Use arrays to show children that 3 x 5 has the same product as 5 x 3. However, subtraction and division do not have this commutative property. Use some counters show children that 12 - 3 is not the same as 3 - 12, and that 15 ÷ 5 is not the same as 5 ÷ 15.

Misconception: multiplication of a number by ten.
Example: in the example 23 x 10 = 230, the pupil believes that to multiply by 10, a zero is ‘added’ to the end of the number.
Solution: this is a logical conclusion to make, as we encourage children to spot patterns and rules. Here, the child has spotted a pattern. The problem with the child’s pattern is that it is not able to be generalised because the rule only works for whole numbers. The child needs to understand that when numbers are multiplied by ten, all the digits move one place to the left. The zero on the end of the number is actually acting as a place holder. Use place value charts and cards and show children that when we multiply by ten or multiples of ten, the number gets ten times bigger. If we were to add a zero to 23 x 10, the number would still be 23.

Misconception: multiplication makes numbers bigger.
Example: 2 x 8 = 16; ½ x 4 = 8.
Solution: children’s early experience of multiplication leads them to believe that multiplication makes numbers bigger, as multiplying any positive number by a whole number greater than one will always increase its value (e.g. 2 x 8 = 16). However, multiplying can have a reducing effect when multiplying a positive number by a fraction less than one. Instead of using the word ‘times’ in the number sentence, use the word ‘of’. For example, ½ of 4 = 2 (not 8).

Misconception: division makes numbers smaller.
Example: 3 ÷ ¼ is the same as 3 ÷ 4 which is equivalent to 0.75.
Solution: children are often familiar with division problems that result in the answer being smaller (e.g. 20 ÷ 5 = 4). With the example above ask children to think of 3 ÷ ¼ as the number of ¼ ‘s that fit into three. There are four quarters in one, so in three there are 3 x 4 quarters which is 12. Alternatively, represent this problem visually.

1		1		1	
1/4	1/4				
1/4	1/4				

Misconception: division is only seen as sharing and not grouping.
Example: 21 ÷ 3 = ? Children share 21 marbles between three friends so that they all receive seven marbles each. However, they could have also taken away three from 21, until there were no more groups of three left to take away, and ended up with seven piles of three.
Solution: recognising division as sharing is the structure of division that both adults and children most naturally associate with this operation. Sharing refers to a situation where a quantity is shared out equally into a given number of portions, and we are asked to determine how many there are in each portion. In the above example, the question is ‘How 21 marbles might be shared equally between three friends? However, there are other structures that we can use to solve division calculations. Grouping is a different way of thinking about a division question. In the example

above, the question now asked is ‘how many groups of three are there in 21?’ Both division structures result in the same answer, although they are answering two different questions! Give children division questions and ask them to work out the answers first by using the ‘sharing’ structure and then by using the ‘grouping’ one. This will help children to understand that both structures result in the same answer. It is also useful to use concrete objects, such as counters to demonstrate this. Take two piles of 20 counters. Write the division question 20 ÷ 4 on the board. Model how to use the sharing structure first by drawing four circles and then place counters in each circle equally until there are no more left. Now take the other pile of counters. This time, take four counters away, then another four and continue until there are no more left. This will allow children to see that while the answer is the same (20 ÷ 4 = 5) the approach to solving it is different.

Misconception: the decimal point moves rather than the digits.
Example: a pupil has been asked to convert measures in metres to centimetres (e.g. 1.34m = ?cm) . When asked how they worked it out, they explained they moved the decimal point forwards twice.
Solution: children need to understand that the digits move (not the decimal point). Reinforce that digits move to the left when multiplying by powers of ten, and to the right when dividing by powers of ten. Use place value charts and cards to show how digits move around the decimal point.

Misconception: addition of negative numbers.
Example: -8 + 6 = 2.
Solution: children may interpret this calculation as finding the difference between eight and six, resulting in the answer two. However, the correct answer is -2. Explain to children that we do not normally write +6 to represent the number six, but we need to include the negative sign for negative numbers to distinguish between negative and positive integers. Use a number line to show children that starting at -8 and counting on six, you end up at -2. Number lines can either be represented horizontally or vertically. While there are several contexts we can use to show negative numbers, the most familiar is probably temperature, which uses a vertical number line.

Misconception: subtraction of negative numbers.
Example: 6 - -3 = 3.
Solution: the misconception here is that subtraction means take away. This structure only applies to positive numbers; you cannot ‘take away’ a negative number. The concept of using six concrete objects to ‘take away’ three negative objects does not work. However, if we think about subtraction as finding the difference, then we can understand that 6 - -3 = 9. Use a number line to model this to children. Mark on the number 6 and the number -3. Ask ‘What is the difference between these two numbers?’ (9).

Algebra

Misconception: continuing a number pattern.
Example: in the number sequence 4, 7, 10, 13, 16, a pupil writes the next number as 18 (not 19).
Solution: this could be a mistake or a misconception. The children may simply have miscounted. They may also have focused in on the last number – 16 – and thought that the pattern is related to even numbers. However, they could also have misinterpreted the symbolic representation of numbers and perhaps have given a random response, which happens to be 18, but could be any other number.

Misconception: using the equalities sign more than once in a number sentence incorrectly.
Example: 28 + 5 = 33 – 8 = 25.
Solution: remind the children that the equals sign is not simply a device for connecting the calculation they have just worked out to the answer. Often children include more than one equals sign in a number sentence because it represents their thinking. In the example above children may have been given the problem 'You have 28 cubes, you are given five more but then eight are taken away. How many cubes do you now have?' In working out the answer, they have added 28 and five cubes together to total 33, and then subtracted eight to reach 25. However, mathematically this is incorrect as 28 = 5 does not equal 33 – 8. There are numbers sentences however, that do use more than one equals sign, for example, 5 + 3 = 3 + 5 = 8. In this number sentence, each section equals eight and therefore, if we return to the image of balance scales, is mathematically correct.

Misconception: finding an unknown in a number sentence.
Example: in the number sentence 2 + □ = 5, a child writes 7 in the empty box.
Solution: children are usually very familiar with number sentences such as 2 + 3 = ? so when they see an addition sign, their instinct is to add the two numbers together. Children need to understand that numbers either side of the equals sign need to balance. Using balance scales can help children to see the relationship between either side of the equals sign. In this example, ask children what number do we need to add to two to equal five (3).

Misconception: when solving number sentences with a variety of operations in them children end up with different answers.
Example: 3 + 5 x 2 = 16 or 3 + 5 x 2 = 13.
Solution: explain to children that there is a correct way to solve these types of questions that is essential to follow to avoid ambiguity and confusion. The order in which operations should be performed if brackets are not used are division or multiplication as it occurs from left to right, addition or subtraction, as it occurred from left to right.

Misconception: the concept of a variable.
Example: the class are working out how many pounds are equivalent to euros. To extend the class the teacher asks how many euros he would get for n pounds. One pupil replies that this is impossible as you can't have n pounds.
Solution: the central concept in algebra is that letters can be used to represent variables which enable us to express generalisations. In the example above the child does not understand the concept of a variable and that n can be used to represent any number of Dinars. It is useful to begin children's algebraic thinking by first being to express patterns in numbers using words first and then in symbols. Start simply. For example, write down the numbers one to five in a table. In the second column write

down numbers that you have applied a rule to. Children usually think that the rule is adding two. This helps if we continue the pattern vertically, but ask children to think what the rule could be if we look at the numbers horizontally. After several examples, ask pupils what number would go with the letter n? Establish that the rule is ‘double and add one’. Now we know the rule, we can see that using a letter to stand for any number (a variable) helps us to express this problem generally.

A	B
1	3
2	5
3	7
4	9
5	11

Misconception: a variable always has the same value.
Example: x always equals 2.
Solution: explain to children that while a variable is often represented by the letter x, other letters can be used too. A variable stands for an unknown number or quantity, and can represent anything. Give children problems where the variables have different letters and quantities.

Misconception: thinking that a variable represents an object rather than a number.
Example: if there are d days in w weeks, then w = 7d because a week equals seven days.
Solution: this misconception usually arises because children are given problems to solve which use the letters as shorthand for the words. For example, d is written instead of days. While the d is being used as an abbreviation for days, it is misleading children to think that this is what letters represent in algebra – which they do not. Explain to children that letters do not represent objects but that they are variables. Give children problems where the letters do not correspond to any of the words in the question, or ask the children to suggest what letter they would like to use with this problem.

Misconception: expanding brackets incorrectly.
Example: 5 (x + 2) = (5x + 2).
Solution: algebraic expressions can often be written in different ways, and while they mean the same this often causes misconceptions for children if they do not understand. Brackets are used to group terms together. The term in front of the bracket can be multiplied by each term in the brackets. The example above should read 5x + 10.

Misconception: when multiplying x’s together, they are added not multiplied.
Example: x x x = 2 x.
Solution: children can get confused when calculating with letters not numbers. Misconceptions with algebra can often be overcome by substituting numbers back in where the letters currently stand. In the example above, give the x’s a value (e.g. 5). Now multiply the sentence using the numbers and compare the two answers. 5 x 5 = 25, not 2 5.

Measurement

Misconception: using rulers or tape measures to measure but answers are not consistent.
Example: the children are asked to measure the same objects using rulers. When they have finished they all have different answers.
Solution: it is important to show the children how to measure using a ruler or a tape measurer. Often, rulers have a space before the numbers start, but children use the whole length of the ruler to measure. When using tape measurers, children forget about the metal end and include it. It is also important to talk about which scale on the ruler or tape measurer they are using (often rulers and tape measurers have two scales on them).

Misconception: confusion over the conservation of mass.
Example: two balls of Plasticine™ are rolled out and look the same. The children agree they have the same mass. The teacher now takes one of the balls and flattens it. The children now think that because the ball has changed shape the mass has also changed.
Solution: children often believe that changing the shape of an object also affects its mass. Show the children that the ball can be converted back into the original ball, demonstrating that its mass must still be the same.

Misconception: mass and weight are thought to be the same thing.
Example: children are asked to find the weight of an apple, instead of its mass.
Solution: the mathematical meaning of these words is often different to the everyday meanings they have. Explain to the children that mass is the amount of matter within an object compared to weight which is the force of gravity acting upon an object. When we use the word ‘weight’ we often mean ‘mass’ and when we say ‘weigh’ we mean ‘find the mass’. In order for children to understand the mass of objects, you need to use a balance that compares mass. For example, ask children to find objects that have the same mass as 10 pebbles or cubes. Equally, when you ask children to hold items in their hands or use scales, remember that they are experiencing weight and not mass.

Misconception: bigger objects are always heavier.
Example: the teacher shows the children two boxes, one which is large and one which is small. When asked which box is the heaviest, the children point to the larger box. However, it is the smaller box which is the heaviest.
Solution: this misconception is formed because children think that the mass of an object is determined by its volume. Discuss with the children that visual weight does not reflect weight. Show this by finding the mass of the two boxes.

Misconception: volume and capacity are often thought to be the same thing.
Example: the children are asked to calculate the capacity of a solid 3-D shape, such as a cube.
Solution: teach the children that volume is the amount of 3-D space which is occupied by an object compared to capacity which can be defined as a measure of the space with which a 3-D object can be filled. Show the children 3-D solid shapes such as cubes and cuboids. We can calculate the volume these shapes have, but we cannot fill them with anything as they are solid. We usually use the term volume for shapes that are solid. Ask the children to build solid shapes using centimetre cubes. This will enable them to work out the volume of the shapes and understand this mathematical word correctly.

Misconception: conservation of area is not secure.
Example: two shapes that look different but have the same surface area.
Solution: Give the children pieces of paper that are the same size. Ask them to cut them up and rearrange but in a different way. This will show children that just because the shapes now look different, they still have the same surface area.

Misconception: if you double the length of sides on a shape, the area will double too.
Example: if a 3 x 3 square has an area of 9 cm² a 6 x 6 square will have an area of 18cm².
Solution: in number operations, children are familiar with whatever happens to one side of an operation, also happens to the other. Therefore, it is common for children to think that if they double the length of the sides on a shape the area will also double. In the example above, give children squared paper, and ask them to draw the 3 x 3 square on the paper and work out the area (9cm²). Repeat with the 6 x 6 square. According to the children's logic, the area of the second square will be 18cm². However, using squared paper the children can calculate that it is 36cm².

Misconception: understanding the difference between area and perimeter.
Example: a shape has an area of 4cm² and a perimeter of 10cm. Children are confused and reverse the area and perimeter measurements.
Solution: explain to children that area is the amount of two-dimensional space inside a boundary compared to perimeter which is the length of the boundary. Children often think there is a relationship between perimeter and area, but there is not. Give children lots of practice in solving questions involving both area and perimeter simultaneously. This will help children to understand the two mathematical concepts.

Misconception: the value of money.
Example: a pupil is given the following coins: 5p, 10p, 20p. The teacher asks him how much money he has. The child says he has 3p. The child has counted the number of coins they have instead of finding the total value of the coins. He has counted the coins as if they were objects.
Solution: children need lots of play and practise at coin recognition. Set up role play areas where they can pretend to buy things with real coins and work out how much change they will receive.

Misconception: using coins that do not exist.
Example: The teacher asks the children to find as many ways as possible of making 25p. One pupil records one 25p coin. The pupil has included a coin that do not exist in our currency.
Solution: the child has over-generalised number concepts. She has used her knowledge of numbers to find solutions to the problem, but would appear to have insufficient experience of handling real money. Provide lots of opportunity to use real money in the classroom.

Misconception: reading digital times as decimal numbers.
Example: when looking at a bus timetable, children calculate that the time to get from one town to the next will be 2.23. When the teacher asks them how long the journey will take, they reply 'two point two three hours'. The pupils have interpreted their answer as if it were a decimal rather than a length of time.
Solution: discuss with the children that their answer means two hours and 23 minutes. Use an analogue clock and discuss how many minutes there are in one hour, two hours and so on. Now turn the hands on the clock

Misconception: the longer the lines, the bigger the angle.
Example: the teacher shows children two angles. The first angle is an acute angle where the two lines are long, compared to the second angle which is an obtuse angle where the two lines are short. When asked which angle is bigger, children reply the angle with the longer lines.
Solution: children have confused the length of the lines with the size of the angle. Give children a protractor and ask them to measure the angles. This will allow the children to see that the second angle is bigger. Alternatively, place the two angles on top of each other and compare visually.

Misconception: it is impossible to have an angle greater than 360º.
Example: a child is asked for an angle greater than 360º. They reply that 360º is the biggest angle you can have.
Solution: this misconception often develops from a lack of experience. Children learn that there are 360º in a circle and they also have many opportunities to measure and create angles that are smaller than 360º. It is reasonable for them to assume that 360º is the largest angle there is. To overcome this, give them problems which result in answers bigger than 360º. For example, ask them to calculate how many degrees the big hand has moved from one o'clock to five o'clock.

Geometry

Misconception: 2-D shapes that are not presented in the same way each time are often thought to be a different shape or have a different name.
Example: children think a square on its base and a square on a vertex are different shapes.
Solution: children who have been exposed to typical images of shapes and have not explored them in different rotations will focus on the visual image they have of that shape. This leads to difficulties in identifying the same shape presented in different ways. Use concrete examples of the shapes and show children that just because a shape is presented in a different orientation, it is still the same shape. Hold up a square on its base, and then turn it round so that it is on its vertex. Show children that it is still a square, but a different way round.

Misconception: using mathematical language but thinking of its everyday meaning.
Example: when asked to identify the faces of objects such as cubes and cuboids, children reply there are none.
Solution: much of the language we use in mathematics is also used in everyday life too. This is particularly common when teaching geometry. Words such as face and solid are difficult for children to understand in a mathematical context when they are often used at home. It is important to explain to children that we are using these words in a mathematical way. Highlight the different meanings that words can have in everyday and mathematical situations so that children can recognise both definitions.

Misconception: any 2-D shape is a polygon.
Example: children are shown images of a square, a triangle, an oblong and a semi-circle. They describe them all as polygons.
Solution: children are often told that shapes such as squares and triangles are all polygons. When they are presented with other shapes they know, such as a semi-circle, they assume that other 2-D shapes are also polygons. They need to be taught that a polygon must be a closed, plane shape made up of straight lines.

Misconception: a circle only has one side.
Example: a pupil states that a circle only has one side.
Solution: explain that a circle has an infinite number of sides. If you take one section of the circle, it will look curved but as you zoom in, it will look less curved, and eventually you will have linearity. It is a hard concept for children to understand but one that is mathematically correct.

Misconception: definitions of quadrilaterals.
Example: a pupil classifies a square as a square but not a rectangle.
Solution: it is important that children recognise that a square is a particular example of a rectangle. Explore the properties of quadrilaterals with children. Ensure that correct mathematical language is used to help children understand the difference between the set of quadrilaterals, as well as physical representations of the shapes too.

Misconception: line symmetry is the same as reflecting one half of a pattern.
Example: children are asked to mark all the lines of symmetry on a square and an oblong. The children mark four lines of symmetry on each shape.
Solution: sometimes it is useful to link symmetry to fractions. For example, early experiences of symmetry for children can be paintings where children paint one half of the piece of paper, and then fold it to make the painting symmetrical. However, this can also lead to misconceptions about line (or reflective) symmetry. In the example

where children think an oblong has four lines of symmetry, cut out the shape and use rotation from the perpendicular to show the children that an oblong only has two lines of symmetry.

Misconception: shapes are inaccurately reflected in a diagonal line.
Example: two right angles triangles correctly reflected as though line was vertical, but it is a diagonal line.
Solution: children find reflecting with lines that are not horizontal or vertical more difficult. When lines are diagonal, discuss with the children that each point in the reflected shape must have the same perpendicular distance form the line of symmetry. Experiment with cutting the shapes out (once they have been reflected) and folding along the line of symmetry. The two halves should match exactly if the shape has been reflected correctly.

Misconception: the order of rotation is determined by the number of sides an object has.
Example: a regular pentagon is shown to the children. They decide it has an order of rotation of five (not one).
Solution: cut out shapes and physically rotate them around the centre of rotation. This will help children to visualise how many times shapes can be rotated and give them a better understanding of rotational symmetry.

Misconception: tessellation is a repeating pattern.
Example: children believe they have tessellated circles because they have created a repeating pattern.
Solution: explain to the children that tessellation only occurs when shapes fit together without any gaps. However, children sometimes think that only one shape can tessellate; this is called regular tessellation. There is another type of tessellation called semi-regular tessellation. This is where different shapes are combined but there are still no gaps left between the shapes (e.g. squares and octagons semi-tessellate). Ask children to choose different shapes and explore if they tessellate or not.

Misconception: co-ordinates can sometimes be written the wrong way round.
Example: with the co-ordinates (4, 2) the 4 is plotted on the y axis when it should be plotted on the x axis and the 2 on the x axis when it should be on the y axis.
Solution: explain to children that the convention of labelling co-ordinates is that the first co-ordinate is plotted on the x-axis and the second co-ordinate is plotted on the y-axis.

Statistics

Misconception: children pose inappropriate questions to investigate.
Example: one pupil poses the question: 'How many toys do I have?'
Solution: discuss with the children that this is an inappropriate question to ask as once the toys have been counted, there is nothing left to be interpreted. Remind children that data handling is about making sense of the situation. The above example does not need any investigation once the question has been answered. However, it could be extended to investigate who has the most wooden toys, for example.

Misconception: incorrect answers to questions posed about data in a graph or chart.
Example: in a block graph which represents the number of counters different children have, when asked how many more Hannah had than Izzy, children just read off the number of counters Hannah has, instead of working out the difference between the two amounts.
Solution: ask children to look again at the question. It may be necessary to underline the key words in the problem to draw children's attention to the words 'how many more'.

Misconception: collecting data incorrectly.
Example: children are asked to design a grid to collect the ages of children from two classes. The boxes include a space for 8-9 year olds and a space for 9-10 year olds.
Solution: ask children which box you tick if you are 9 years old? Encourage the children to realise that their data will be inaccurate as both boxes could be ticked, representing two people when actually this should only represent one person.

Misconception: there are no gaps between columns on a block graph when the data is discrete.
Example: children draw a block graph where the bars are touching; e.g. goals scored by 5 different football teams.
Solution: remind the children that the word 'discrete' means separate. Discrete data is about particular data which automatically sorts into quite distinct, separate groups. Discuss an example of data that is discrete (for example, goals scored in a football match). Show children that discrete data means there is no connection between the data, compared to continuous data which is linked (e.g. temperature). Ask 'Can we have two and half goals scored in a match?' (no) 'Can we have two and half degrees of temperature?' (yes). Leaving gaps between the columns models pictorially the way the data has sorted itself into separate groups.

Misconception: different widths are used on a block graph.
Example: children draw a bar chart showing different flavour crisps, but using different widths for each flavour.
Solution: explain to children that it is essential that the columns are drawn with equal widths. This produces an appropriate pictorial representation of the data.

Misconception: interpreting units always as one on a chart instead of reading the scale correctly.
Example: in a pictogram showing different ways children travel to school each picture represents two, but children read them as one each.
Solution: this misconception is often derived because children first see data represented in ones. In this example the children simply assume that each picture represents one. Often asking children to create data where they need to use one

picture or unit to represent more than one quantity helps the children to interpret data more accurately next time.

Misconception: axes on a graph are labelled incorrectly.
Example: the axis of a graph starts at one and not zero.
Solution: discuss how the axis is a number line is a scale of measure. Children can become confused because we start counting at one, but forget that we measure from zero.

Misconception: both axes are labelled in the same way.
Example: on a scatter graph, children are recording their shoe size and their height, but they have labelled the graph using the same scale.
Solution: children incorrectly think that they must use the same scale on both axes. Show them alternative graphs and charts that use different scales. Talk about the reasons why different scales are necessary in recording data sets.

Misconception: labels are left out on axes.
Example: children draw a graph with no labels on it (e.g. it could show how many children have which pets)
Solution: ask children to explain what the graph above represents. While this is an interesting debate, it highlights the significance of including labels on graphs and charts as an absence of labels renders the representation meaningless. The key approach here is to discuss the data with the children so that they can see this for themselves.

Misconception: confusion over mode, mean and median.
Example: the children are given a set of numbers, e.g. 5,6,7,4,7,9,7,4,5. They are asked to calculate the mode, and give the answer as six.
Solution: the children have confused the mode and mean. The mode is the most frequently occurring value (7). The mean is the total of the number of items in the set, divided by the number in the set ($54 \div 9 = 6$). When solving problems involving mode, median and mean, make sure that children can see the definitions of each word. It is often hard to remember which word means which.

Misconception: probability.
Example: when a dice is tossed four times and each time it has landed on its head, children think it is unlikely to land on its head next time.
Solution: discuss with children that each toss of the coin is independent to the previous turns. The coin can still land on either heads or tails. Carry out the experiment and show children that it has an equal chance of landing on either the head or tail with each toss.